Closing Today: 2.1, 2.2, 2.3
Closing Tuesday: 2.5-6
Closing next Fri: 2.7, 2.7-8
Extended office hours today 1:30-3pm
in Com. B-006 (next to MSC)

Entry Task: **From HW**, evaluate:

1. $\lim\limits_{t\to\pi/2}\left[\dfrac{\sin(t)+\sqrt{\sin^2(t)+3\cos^2(t)}}{2\cos^2(t)}\right]$

2. $\lim\limits_{t\to\pi/2}\left[\dfrac{\sin(t)-\sqrt{\sin^2(t)+3\cos^2(t)}}{2\sin^2(t)}\right]$

3. $\lim\limits_{t\to\pi/2}\left[\dfrac{\sin(t)-\sqrt{\sin^2(t)+3\cos^2(t)}}{2\cos^2(t)}\right]$

2.5 Continuity (continued…)

A function, $f(x)$, is **continuous at x = a** if

$$\lim_{x \to a} f(x) = f(a)$$

i.e. the following must be equal:

(i) $\lim_{x \to a^-} f(x)$

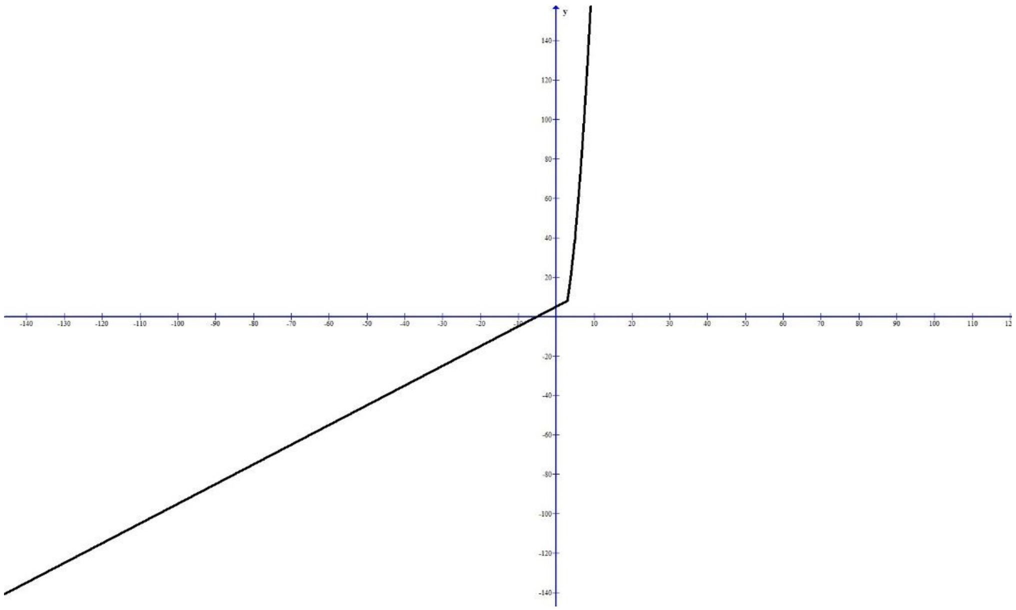
(ii) $\lim_{x \to a^+} f(x)$

(iii) $f(a)$

Example: Find the value of *c* that makes the function continuous everywhere:

$$f(x) = \begin{cases} \dfrac{(x+1)^2 - 16}{x-3} & \text{, if } x < 3; \\ 2x^2 + c & \text{, if } x \geq 3. \end{cases}$$

f(x)

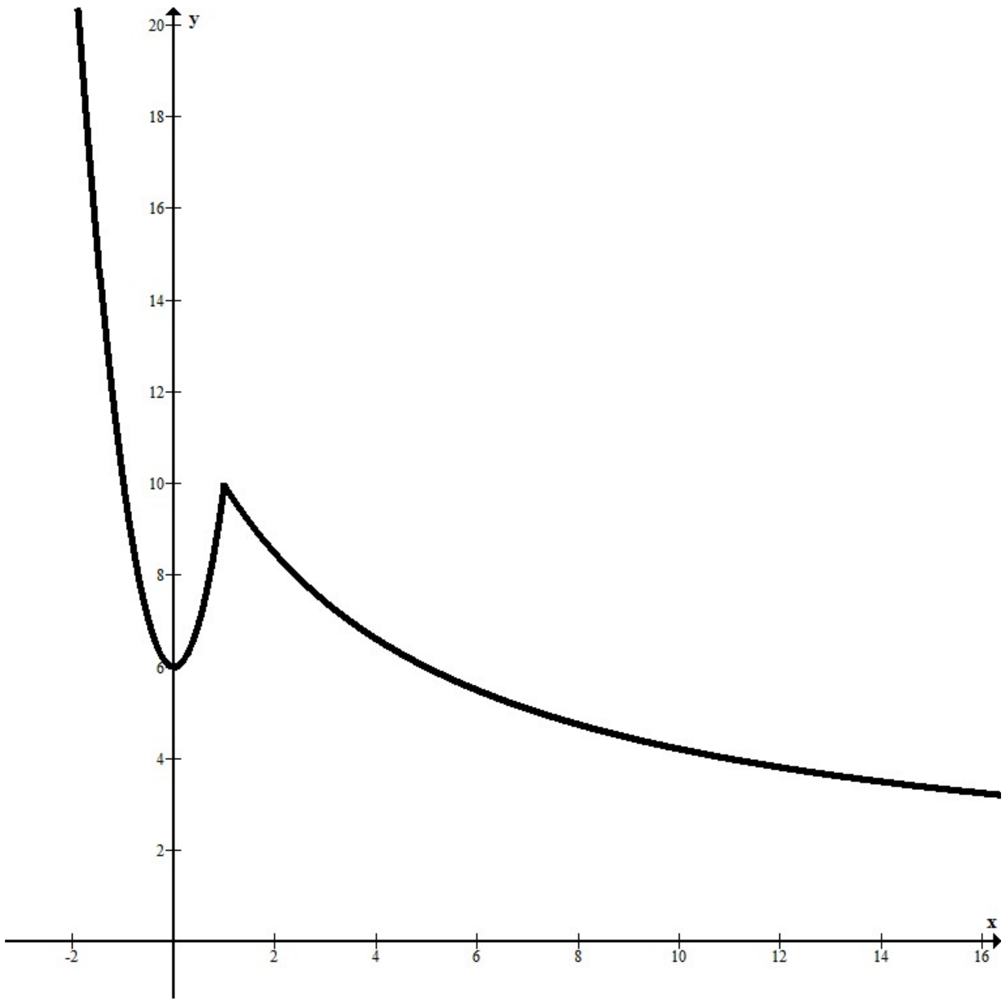


Example:

$$h(x) = \begin{cases} ax^2 + 6 & ,\text{if } x < 1; \\ b & ,\text{if } x = 1; \\ \dfrac{x + 49}{x + a} & ,\text{if } x > 1. \end{cases}$$

Find the values of *a* and *b* that will make $h(x)$ continuous *everywhere*.

h(x)



For 8 more continuity problems like these, see my online practice sheet: “Continuity Practice Problems” (There are posted solutions as well).

Theorem:

If $f(x)$ is continuous at $x = b$, and

$$\lim_{x \to a} g(x) = b$$

then

$$\lim_{x \to a} f\big(g(x)\big) = f(b).$$

Example:

Find

$$\lim_{x \to 9} \ln\left(\frac{\sqrt{x} - 3}{x - 9}\right)$$

2.6 Limits "at" Infinity
(*Horizontal Asymptotes*)

Goal: Study "long term" behavior.

$$\lim_{x\to\infty} f(x) = L$$

"the limit of $f(x)$, as *x* goes to infinity is *L*",
as *x* takes on larger and larger positive numbers,
$y = f(x)$ takes on values closer and closer to L.

Similarly,

$$\lim_{x\to-\infty} f(x) = L$$

"the limit of $f(x)$, as *x* goes to negative infinity is L".

Important limits to know:

1. For any <u>positive</u> number *n*,

$$\lim_{x\to\infty} x^{-n} = \lim_{x\to\infty} \frac{1}{x^n} = 0.$$

$$\lim_{x\to-\infty} x^{-n} = \lim_{x\to-\infty} \frac{1}{x^n} = 0.$$

$$\text{(if defined)}$$

2. $\lim_{x\to\infty} e^x = \infty$ and $\lim_{x\to\infty} e^{-x} = 0.$

$\lim_{x\to-\infty} e^x = 0$ and $\lim_{x\to-\infty} e^{-x} = \infty$

3. $\lim_{x\to\infty} \ln(x) = \infty$

4. $\lim_{x\to\infty} \tan^{-1}(x) = \frac{\pi}{2},$

$\lim_{x\to-\infty} \tan^{-1}(x) = -\frac{\pi}{2}$

Strategies to compute $\lim\limits_{x\to\infty} f(x)$

1. Is it a standard one from my list on the last page?
   If so, *done*. If not, go to next step.

2. Combine into one fraction.

3. **Use algebra to *rewrite* in terms of known limits from previous page**:

Strategy 1: Multiply top/bot by $\frac{1}{x^a}$, where a is the largest power.

Strategy 2: Multiply top/bot by $\frac{1}{e^{rx}}$.

Examples:

1. $\lim\limits_{x\to\infty} \dfrac{3+x^4-x}{4x^2+1-6x^4}$

2. $\lim\limits_{x\to\infty} \left(\dfrac{x}{x+2} - \dfrac{1}{x}\right)$

3. $\lim_{x\to\infty} \dfrac{3+5e^{(2x)}}{2e^{x}+4e^{(2x)}}$

5. $\lim_{x\to-\infty} \dfrac{\sqrt{9x^6-x+1}}{2x^3-x^2}$

Note: $\sqrt{x^2} = \quad x, \text{if } x \geq 0,$ and

$\sqrt{x^2} = -x, \text{if } x < 0.$

4. $\lim_{x\to\infty} \dfrac{\sqrt{9x^6-x+1}}{2x^3-x^2}$

5. $\lim\limits_{x \to \infty} \left(\sqrt{3 + 2x + x^2} - x \right)$

Strategies to compute: $\lim\limits_{x \to a} \left[\frac{f(x)}{g(x)}\right]$

Special note: If given two fractions, combine them (common denom).

Try plugging in the value:

1. **If denominator ≠ 0, done!**
2. **If denom = 0 & numerator ≠ 0**, the answer is -∞, +∞ or DNE. Examine the sign of the output from each side.
3. **If denom = 0 & numerator = 0**, Use algebra to simplify and cancel until either the numerator or denominator is not zero.

Strategy 1: Factor/Cancel
Strategy 2: Simplify Fractions
Strategy 3: Expand/Simplify
Strategy 4: Multiply by Conjugate
(if you see radicals)

Strategies to compute: $\lim\limits_{x \to \infty} f(x)$

Special note: Combine into one fraction (might need conjugate if given two terms involving a radical).

1. Is it a known limit?

$$\lim_{x \to \infty} \frac{1}{x^a} = 0, \text{if } a > 0; \quad \lim_{x \to \infty} e^{-x} = 0;$$

$$\lim_{x \to \infty} \ln(x) = \infty; \quad \lim_{x \to \infty} \tan^{-1}(x) = \frac{\pi}{2}.$$

2. Rewrite in terms of known limits:

Strategy 1: Multiply top/bottom by $\frac{1}{x^a}$,
where *a* is the largest power.
Strategy 2: Multiply top/bottom by e^{-rx}.

Special note:

If *x* is positive, then $x = \sqrt{x^2}$.
If *x* is negative, then $x = -\sqrt{x^2}$.